Modalized questions and exhaustivity

Consider the contrast between the two following question-answer pairs (cf. Fox & Hackl 2005):

(1) a. What books did Jack read?  b. Jack read more than two French novels.
>> b. doesn’t imply that Jack didn’t read more than three French novels.

(2) a. What books must Jack read?  b. Jack must read more than two French novels.
>> Implicature of b: Jack is not required to read more than three French novels.

I argue that this contrast follows from a) the interpretation of wh-questions that contain a modal, which motivates a revision of standard assumptions, and b) a new account of the phenomenon of exhaustive interpretation of answers.

I. Wh-phrases can bind $<<e,t>,t>$-variables. Suppose that among books A, B, C, etc, Jack has to read either both A and B or both C and D (with ‘either…or’ interpreted within the scope of ‘has to’), and has no other reading obligation. Consider then:

(3) What books does Jack have to read?

The two following answers could be uttered by a cooperative and fully informed speaker:

(4) a. No book in particular        b. Both A and B or both C and D

(4)a is expected from the point of view of standard accounts (Karttunen 1977, Groenendijk & Stockhof 1982) of the interpretation of wh-questions, according to which (3) asks for a specification of the extension of the predicate $\lambda y (y$ is a book & Jack has to read $y$), which turns out to be empty in the above scenario (since there is no particular book that Jack must read). In contrast with this, (4)b, which is understood as $Jack has to read either both A and B or both C and D$, is not predicted to count as a complete or even a partial answer in such accounts. I propose that besides the reading predicted by standard accounts, (3) has a reading corresponding to the following informal logical form, which underlies the answer in (4)b:

(5) For which generalized quantifier $Q$ over books, does Jack have to read $Q$? [details below]

That we are dealing here with a genuinely distinct reading, and not with a purely pragmatic fact, is shown by a) the interpretation of questions like (3) in embedded contexts, and b) the fact that this type of reading is sensitive to weak islands.

a. Interpretation of embedded questions

(6) Mary guessed what books Jack has to read

In the above scenario, (6) can be deemed false if Mary only guessed that there is no book in particular that Jack has to read, without guessing that he has to read either both A and B or both C and D. Following most accounts of the semantics of embedded questions (Karttunen 1977, Groenendijk & Stockhof 2002, Heim 1994, Beck & Rullman 1999, and many others), I assume that (6) states that Mary is in the relation denoted by guess to the actual complete answer to the embedded question; it follows the complete answer to What books does Jack have to read?, in the above scenario, must entail or be equivalent to $Jack has to read either A and B or C and D$.

b. Weak-island effects

(7) a. What books must Jack read?  b. What books does Mary know that Jack read?

(8) Either two books by Balzac or two books by Joyce

(8), if given as an answer to (7)a, is naturally understood as taking narrow scope under must. On the other hand, (8) cannot be interpreted as taking scope below know if given as an answer to (7)b. Rather, the only interpretation in this case is the wide-scope one: either there are two books by Balzac such that Mary knows that Jack read them, or there are two books by Joyce such that Mary knows that Jack read them. This suggests that the following logical form is not available:

(9) For which generalized quantifier $Q$ over books, is it true that Mary knows that Jack read $Q$?

This type of reading is thus sensitive to factive islands (and, in fact, to other types of weak islands).
c. Proposal. Wh-phrases are ambiguous, and have (at least) the two following readings:

\[(10)\]

a. \[ \{ (What books) \ (S) \}^w = \lambda \phi_{\in \{books\}^w} \exists x \in \{books\}^w (x \in (S)^w \land \phi) = \{ [S] \to x \land \phi(w) = 1 \} \]

b. \[ \{ (What books) \ (S) \}^w = \lambda \phi_{\in \{books\}^w} G_{\in \{books\}^w} (G \text{ is a monotone-increasing GQ whose smallest live-on set belongs to } \{books\}^w \land \phi = \{ [S] \to G \land \phi(w) = 1 \}) \]

\[ \text{with } \alpha \text{ a variable index ranging over generalized quantifiers; the restriction to increasing GQs is meant to capture certain asymmetries between “positive” and “negative” answers – The notion of smallest live-on set can be found in Szabolcsi (1997); for any conservative determiner } D \text{, the smallest live-on set of } [D \text{ NP}] \text{ is the denotation of NP.} \]

According to (10)a, the denotation of (1)a is, as in Karttunen (1977), the set of all true propositions of the form Jack has to read x, with x standing for an individual, atomic or plural. According to (10)b, it is the set of all true propositions of the form Jack must read G, with G an increasing generalized quantifier over books.

II. Exhaustivity in modal contexts. Given a question Q, I define the complete answer to Q in a world w as the logically strongest member of the denotation of Q in w. In many cases, this definition is equivalent to Karttunen’s (1977) definition (conjunction of all the true elementary answers), once we adopt an ontology with plural individuals. If no operator intervenes between the wh-phrase and its extraction site, the complete answer so defined is, in any world, the same whether we use (10)a or (10)b. But in the case of (3), the complete answer (in the relevant scenario) is (4)a on the basis of (10)a, and (4)b on the basis of (10)b. Next, I define an operator, OP_Q, which, when applied to an answer S to a question Q, states that S is the strongest member of the denotation of Q:

\[(11)\] a. Who came?  
\[ b. \text{Peter and Mary came} \]

\[(12)\] \[ \{[OP_{(11)a}(Peter and Mary came)]^w = 1 \text{ iff 'Peter and Mary came' is the strongest true member of } \{[(11)a]^w \}, \text{i.e. if Peter and Mary came and nobody else did in w.} \]

However, applying OP_{(11)a} to disjunctive or indefinite answers generally yields a contradiction, because in no world w do such answers express propositions that belong to the denotation of (11)a. In such cases, I assume OP_Q can occur in an embedded position (cf. also Keshet 2006):

\[(13)\] a. (Three linguists) (\lambda x.OP_{(11)a}(x came))  
\[ b. \text{(More than two linguists) } (\lambda x.OP_{(11)a}(x came)) \]

These two logical forms are equivalent to (respectively):

\[(14)\] a. \( \exists X (\text{linguists}(X) \land \#X=3 \land OP_{(11)a}(X \text{ came}), \text{i.e. There is a group consisting of exactly three linguists such that the proposition that this group came is the complete answer to (11)a, i.e. Exactly three linguists came and nobody else did} \)

b. \( \exists X (\text{linguists}(X) \land \#X>2 \land OP_{(11)a}(X \text{ came}), \text{i.e. There is a group consisting of more than two linguists such that the proposition that this group came is the complete answer to (11)a, i.e. More than two linguists came and no non-linguist came} \)

I therefore derive that more than two linguists came, contrary to three linguists came, doesn’t implicate in this context that no more than four linguists came. But consider now:

\[(15)\] a. What books must Jack read?  
\[ b. \text{Jack must read more than two books by Balzac} \]

It turns out that the proposition that (15)b is the complete answer to (15)a is not contradictory, on the reading corresponding to (10)b: (15)b is indeed the complete answer in any world in which Jack’s only reading obligation is to read more than two books by Balzac. As a result, OP_{(15)a}(15)b denotes the proposition that Jack’s only reading obligation is to read more than two books by Balzac, from which it follows that a) Jack may choose any three books by Balzac, and b) Jack is not required to read more than three books by Balzac. I will show that my proposal also sheds light on the interpretation of various scalar items within the scope of possibility modals.